A Persymmetric GLRT for Adaptive Range Spread Target Detection in Non-Homogeneous Environment

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RESEARCH QUESTION

The problem of range spread target detection embedded in subspace interference and compound Gaussian noise with gamma texture is dealt with. The interference belongs to a known subspace but the coordinates are unknown. Exploiting the persymmetric structure of noise covariance matrix, a persymmetric generalized likelihood ratio test is proposed. The performance assessment demonstrates the superiority of the proposed detector.

PROBLEM FORMULATION

Problem Formulation

The received echoes are collected from N channels. We formulate the detection problem as the following binary hypothesis test

$$\begin{cases} H_0: \begin{cases} \boldsymbol{r}_l' = \boldsymbol{J}'\boldsymbol{\varphi}_l + \boldsymbol{n}_l', l = 1,\ldots,L \\ \boldsymbol{r}_k' = \boldsymbol{n}_k', k = 1,\ldots,K \end{cases} \\ H_1: \begin{cases} \boldsymbol{r}_l' = \boldsymbol{H}'\boldsymbol{\theta}_l + \boldsymbol{J}'\boldsymbol{\varphi}_l + \boldsymbol{n}_l', l = 1,\ldots,L \\ \boldsymbol{r}_k' = \boldsymbol{n}_k', k = 1,\ldots,K \end{cases} \end{cases}$$

where: $\boldsymbol{r}_l'$ denotes the primary data, $\boldsymbol{r}_k', k = 1,\ldots,K$ denotes the training data, $\boldsymbol{H}' \in C^{N\times p}$ denotes the signal subspace, $\boldsymbol{J}' \in C^{N\times q}$ denotes the interference subspace.

The K distribution is exploited to describe the noise amplitudes since it fits the real data well. Then, the texture component follows the gamma distribution:

$$\boldsymbol{n}_l' = \sqrt{\tau_l}\,\boldsymbol{g}_l, l = 1,\ldots,L$$

$$\boldsymbol{n}_k' = \sqrt{\tau_k}\,\boldsymbol{g}_k, k = 1,\ldots,K$$

$$f(\tau_{l'}) = \frac{1}{\beta_{l'}^{\vartheta_{l'}}\Gamma(\vartheta_{l'})}\tau_{l'}^{\vartheta_{l'}-1}\exp\left(-\frac{\tau_{l'}}{\beta_{l'}}\right)$$

To overcome the performance degradation in the limited-training situations, we exploit the persymmetric structure of the noise covariance matrix:

$$\begin{cases} H_0: \begin{cases} \boldsymbol{r}_l = \boldsymbol{J}\boldsymbol{\varphi}_l + \boldsymbol{n}_l, l = 1,\ldots,L \\ \boldsymbol{r}_k = \boldsymbol{n}_k, k = 1,\ldots,K \end{cases} \\ H_1: \begin{cases} \boldsymbol{r}_l = \boldsymbol{H}\boldsymbol{\theta}_l + \boldsymbol{J}\boldsymbol{\varphi}_l + \boldsymbol{n}_l, l = 1,\ldots,L \\ \boldsymbol{r}_k = \boldsymbol{n}_k, k = 1,\ldots,K \end{cases} \end{cases}$$

where: $\boldsymbol{r}_l = \boldsymbol{T}\boldsymbol{r}_l'$, $\boldsymbol{r}_k = \boldsymbol{T}\boldsymbol{r}_k'$, $\boldsymbol{n}_l = \boldsymbol{T}\boldsymbol{n}_l'$, $\boldsymbol{n}_k = \boldsymbol{T}\boldsymbol{n}_k'$, $\boldsymbol{H} = \boldsymbol{T}\boldsymbol{H}'$, $\boldsymbol{J} = \boldsymbol{T}\boldsymbol{J}'$, $\boldsymbol{\Sigma} = \boldsymbol{T}\boldsymbol{\Sigma}'\boldsymbol{T}^H$,

$$\boldsymbol{T} = \begin{cases} \frac{1}{\sqrt{2}}\begin{pmatrix} \boldsymbol{I}_{N/2} & \boldsymbol{P}_{N/2} \\ j\boldsymbol{I}_{N/2} & -j\boldsymbol{P}_{N/2} \end{pmatrix} & \text{for } N \text{ even} \\ \frac{1}{\sqrt{2}}\begin{pmatrix} \boldsymbol{I}_{(N-1)/2} & 0 & \boldsymbol{P}_{(N-1)/2} \\ 0 & \sqrt{2} & 0 \\ j\boldsymbol{I}_{(N-1)/2} & 0 & -j\boldsymbol{P}_{(N-1)/2} \end{pmatrix} & \text{for } N \text{ odd} \end{cases}, \quad \boldsymbol{P} = \begin{bmatrix} 0 & 0 & \ldots & 0 & 1 \\ 0 & 0 & \ldots & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 1 & \ldots & 0 & 0 \\ 1 & 0 & \ldots & 0 & 0 \end{bmatrix}$$

DESIGN OF THE ADAPTIVE GLRT DETECTOR

Detectors Design

When the noise covariance matrix is known, the GLRT which is used to solve the detection problem is given as

$$\frac{\max\limits_{\Theta,\Phi}\int f(\boldsymbol{R}_L \mid \Theta,\Phi,\boldsymbol{\tau},H_1) f(\boldsymbol{\tau}) d\boldsymbol{\tau}}{\max\limits_{\Phi}\int f(\boldsymbol{R}_L \mid \Phi,\boldsymbol{\tau},H_0) f(\boldsymbol{\tau}) d\boldsymbol{\tau}} \underset{H_0}{\overset{H_1}{\gtrless}} \eta$$

where: $\boldsymbol{R}_L = [\boldsymbol{r}_1,\ldots,\boldsymbol{r}_L], \Theta = [\boldsymbol{\theta}_1,\ldots,\boldsymbol{\theta}_L], \Phi = [\boldsymbol{\varphi}_1,\ldots,\boldsymbol{\varphi}_L], \boldsymbol{\tau} = [\tau_1,\ldots,\tau_L]$.

$f(\cdot|H_i)$ denotes the probability density function of the primary data:

$$f(\boldsymbol{R}_L \mid \Theta,\Phi,\boldsymbol{\tau},H_i) = \prod_{l=1}^{L}\frac{1}{\pi^N\tau_l^N\det(\boldsymbol{\Sigma})}$$

$$\times\exp\left[-\frac{1}{\tau_l}(\boldsymbol{r}_l - i\boldsymbol{H}\boldsymbol{\theta}_l - \boldsymbol{J}\boldsymbol{\varphi}_l)^H\boldsymbol{\Sigma}^{-1}(\boldsymbol{r}_l - i\boldsymbol{H}\boldsymbol{\theta}_l - \boldsymbol{J}\boldsymbol{\varphi}_l)\right]$$

According to the property of the modified Bessel function of the second kind, we have the maximum likelihood estimate of the unknown parameter under null hypothesis:

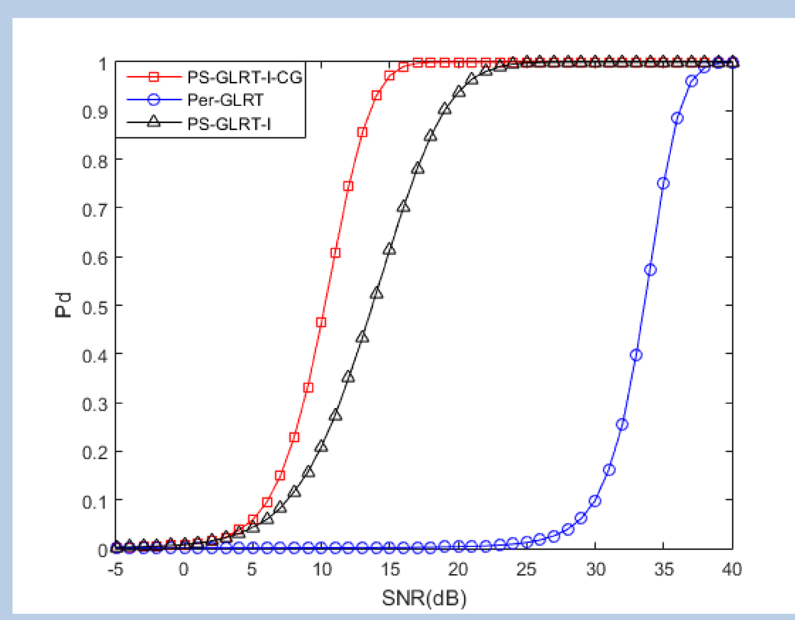
$$\hat{\boldsymbol{\varphi}}_l = \left(\boldsymbol{J}^H\boldsymbol{\Sigma}^{-1}\boldsymbol{J}\right)^{-1}\boldsymbol{J}^H\boldsymbol{\Sigma}^{-1}\boldsymbol{r}_l$$

The MLEs of the unknown parameters under alternative hypothesis can be calculated in a similar way. Since the noise covariance matrix is usually unknown, we substitute the persymmetric fixed point covariance estimator to replace the known one to get the fully adaptive GLRT detector as

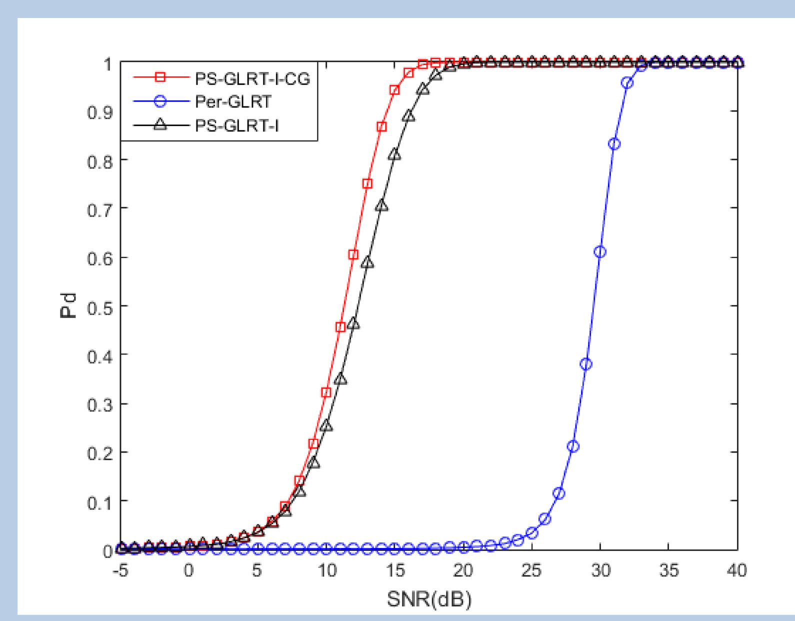
$$\left(\frac{\hat{\boldsymbol{T}}_{1l}}{\hat{\boldsymbol{T}}_{0l}}\right)^{\frac{\vartheta_l - N}{2}}\frac{K_{N-\vartheta_l}\left(2\sqrt{\hat{\boldsymbol{T}}_{1l}/\beta_l}\right)}{K_{N-\vartheta_l}\left(2\sqrt{\hat{\boldsymbol{T}}_{0l}/\beta_l}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta$$

SIMULATION RESULTS

Analysis and Evaluation of Simulation Results



(a) Detection probabilities of the detectors for K=N



(b) Detection probabilities of the detectors for K=2N

- The performance gains of the proposed PS-GLRT-I-CG with respect to the PS-GLRT-I are 6dB and 2dB for K=N and K=2N, respectively. The performance gain of the proposed PS-GLRT-I-CG with respect to the Per-GLRT is more than 10dB.

CONCLUSIONS

The problem of adaptive range spread target detection in subspace interference plus compound Gaussian noise with gamma texture has been discussed in this paper. By exploiting the persymmetric property of the noise covariance matrix and GLRT design rule, the PS-GLRT-I-CG has been proposed. Simulation results show that the PS-GLRT-I-CG outperforms the conventional detectors.